Exercise.13
Formation of ANOVA table for Latin square design (LSD) and comparison of means using critical difference values

Latin Square Design

When the experimental material is divided into rows and columns and the treatments are allocated such that each treatment occurs only once in each row and each column, the design is known as L S D.

In LSD the treatments are usually denoted by A B C D etc.

For a 5 x 5 LSD the arrangements may be

A	***B***	***C***	***D***	***E***
B	***A***	***E***	***C***	***D***
C	***D***	***A***	***E***	***B***
D	***E***	***B***	***A***	***C***
E	***C***	***D***	***B***	***A***

Square 1

A	***B***	***C***	***D***	***E***
B	***A***	***D***	***E***	***C***
C	***E***	***A***	***B***	***D***
D	***C***	***E***	***A***	***B***
E	***D***	***B***	***C***	***A***

Square 2

A	***B***	***C***	***D***	***E***
B	***C***	***D***	***E***	***A***
C	***D***	***E***	***A***	***B***
D	***E***	***A***	***B***	***C***
E	***A***	***B***	***C***	***D***

Square 3

Analysis

The ANOVA model for LSD is

$$Y_{ijk} = \mu + r_i + c_j + t_k + e_{ijk}$$

r_i is the ith row effect

c_j is the jth col effect

t_k is the kth treatment effect

The analysis of variance table for LSD is as follows:

Sources of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-ratio
Rows	t-1	RSS	RMS	RMS/EMS
Columns	t-1	CSS	CMS	CMS/EMS
Treatments	t-1	TrSS	TrMS	TrMS/EMS
Error	(t-1)(t-2)	ESS	EMS	
Total	t^2-1	TSS		

F table value

$F_{[t-1),(t-1)(t-2)]}$ degrees of freedom at 5% or 1% level of significance

Steps to calculate the above Sum of Squares are as follows:

Correction Factor (CF)= $\frac{(GrandTotal)^2}{(treatment)^2}$

Total Sum of Squares (TSS)= $\sum (y_{ijk})^2 - CF$

Row sum of squares (RSS)= $\frac{1}{t}\sum_{i=1}^{t}(R_i)^2 - CF$

Column sum of squares (CSS)= $\frac{1}{t}\sum_{j=1}^{t}(C_j)^2 - CF$

Treatment sum of squares (TrSS)= $\frac{1}{t}\sum_{K=1}^{t}(T_k)^2 - CF$

Error Sum of Squares = TSS-RSS-CSS-TrSS

These results can be summarized in the form of analysis of variance table.

Calculation of SE, SE(d) and CD values

SE = $\sqrt{\frac{EMS}{r}}$

where r is the number of rows

SE(d) = $\sqrt{2}$ x SE

CD= t x SE(d)

where t =table value of t for a specified level of significance and error degrees of freedom

Using CD value the bar chart can be drawn and the conclusion may be written.

Problem

Below are given the plan and yield in kgs/plot of a 5x5 Latin square experiment on the wheat crop carried out for testing the effects of five, manorial treatments A, B, C, D, and E. 'A' denotes control.

B	15	A	8	E	17	D	20	C	17	R1	= 77
A	9	D	21	C	19	E	16	B	13	R2	= 78
C	18	B	12	D	23	A	8	E	17	R3	= 78
E	18	C	16	A	10	B	15	D	23	R4	= 82
D	22	E	15	B	13	C	18	A	10	R5	= 78

$C_1 = 82, C_2 = 72, C_3 = 82, C_4 = 77, C_5 = 80$; GT = 393

Analyze the data and state your conclusions.

Analysis

1. Correction factor = $\frac{(GT)^2}{rxc}$ where GT is the grand total 'r' is number of rows, and 'c' is number of columns

$= \frac{(393)^2}{5x5} = 6177.96$

2. Total SS = $152 + 82 = \ldots.. + 102 - CF$

$= 666_1 - 6177.96 = 483.04$

3. SS due to rows (SSR) $= \dfrac{R_1^2 + R_2^2 + \ldots.. + R_5^2}{t} - CF$

$= \dfrac{77^2 + \ldots..\ldots + 78^2}{5} - 6177.96 = 3.04$

4. SS due to columns (SSC) $= \dfrac{C_1^2 + C_2^2 + \ldots.. + C_5^2}{t} - CF$

$= \dfrac{82^2 + \ldots..\ldots + 80^2}{5} - 6177.96$

$= 14.24$

5. To get SS due to treatments, first find the totals for each treatment using the given data as follows:

Treatment (A)	**B**	**C**	**D**	**E**
8	15	17	20	17
9	13	19	21	16
8	12	18	23	17
10	15	16	23	18
10	13	18	22	15
$T_1 = 45$	$T_2 = 68$	$T_3 = 88$	$T_4 = 109$	$T_5 = 83$

$\therefore$ SS due to treatments $= \dfrac{T_1^2 + T_2^2 + \ldots.. + T_5^2}{r} - CF$

$= \dfrac{45^2 + \ldots.. 83^2}{5} - 6177.96$

$= 454.64$

6. SS due to error = TSS – SSR – SSC – SST

= 483.04 – 3.04 -14.24-454.6 =11.12

7. Table for analysis of variance

Source of variation	Df	SS	MS	Variance ratio F	F value at 5% level & 1% level
Rows	4	3.04	0.76	123.34**	3.26 5.41
Columns	4	14.24	3.56		
Treatments	4	454.24	113.66		
Error	12	11.12	0.92		
Total	24	483.04			

** Highly significant

The observed highly significant value of the variance ratio indicates that there are significant differences between the treatment means.

S.E. of the difference between the treatment means (SED)

$= \sqrt{\frac{2XEMS}{r}}$ where EMS indicates the error mean square and 'r'

indicates the number of replications.

i.e. SEd = $= \sqrt{\frac{2 \times 0.92}{5}} = 0.61$

∴ Critical difference = SEd x t 5% at df = 12 = 0.61 x 2.179

= 1.33

Summary of results

Treatment means will be calculated from the original table on treatment totals.

Treatments	A	B	C	D	E	CD 5%
Mean yield in kgs / plot	9.0	13.6	17.6	21.8	16.6	1.33

Conclusion represented symbolically

The treatment have been compared by setting them in the descending order of their yields.

Treatments : D C E B A

Mean yields In kgs/plot	21.8	17.6	16.6	13.6	9.0

The treatment 'D' is the best of all. The treatments 'C' and 'E' do not differ significantly each other.

The yield obtained by applying every one of the manorial treatment is significantly higher that obtained without applying any manure.

Learning Exercise

1. An oil company tested four different blends of gasoline for fuel efficiency according to a Latin square design in order to control for the variability of four different drivers and four different models of cars. Fuel efficiency was measured in miles per gallon (mpg) after driving cars over a standard course.

Fuel Efficiencies (mpg) For 4 Blends of Gasoline
(Latin Square Design: Blends Indicated by Letters A-D)

	Car Model			
Driver	**I**	**II**	**III**	**IV**
1	**D** 15.5	**B** 33.9	**C** 13.2	**A** 29.1
2	**B** 16.3	**C** 26.6	**A** 19.4	**D** 22.8
3	**C** 10.8	**A** 31.1	**D** 17.1	**B** 30.3
4	**A** 14.7	**D** 34.0	**B** 19.7	**C** 21.6

These data are from Ott: *Statistical Methods and Data Analysis,* 4th ed., Duxbury, 1993, page 866. (*Similar* data are given in the 5th edition by Ott/Longnecker, in problem 15.10, page 889.)

Analyse the data and draw yor conclusion.

2. The numbers of wireworms counted in the plots of Latin square following soil fumigations (L,M,N,O,P) in the previous year were

	Columns				
Rows	P(4)	O(2)	N(5)	L(1)	M(3)
	M(5)	L(1)	O(6)	N(5)	P(3)
	O(4)	M(8)	L(1)	P(5)	N(4)
	N(12)	P(7)	M(7)	O(10)	L(5)
	L(5)	N(4)	P(3)	M(6)	O(9)

Analyse the data and draw your conclusions.

3. The following layout presents the observations made on 5 treatments A,B, C,D and E in an experiment of paddy crop by adopting LSD. The figures indicate the grain yield of paddy in kg/plot. Analyse the data and draw your conclusion.

	Columns				
Rows	B 5	D 6	E 3	A 10	C 12
	C 9	A 4	B 6	E 5	D 5
	D	C	A	B	E

			8	15	7	6	5	
			E 5	B 8	C 13	D 9	A 5	
			A 9	E 6	D 12	C 16	B 8	