INVERSE OF A MATRIX

Definition

Let A be any square matrix. If there exists another square matrix B Such that AB = BA = I (I is a unit matrix) then B is called the inverse of the matrix A and is denoted by A^{-1}.

The cofactor method is used to find the inverse of a matrix. Using matrices, the solutions of simultaneous equations are found.

Working Rule to find the inverse of the matrix

Step 1: Find the determinant of the matrix.

Step 2: If the value of the determinant is non zero proceed to find the inverse of the matrix.

Step 3: Find the cofactor of each element and form the cofactor matrix.

Step 4: The transpose of the cofactor matrix is the adjoint matrix.

Step 5: The inverse of the matrix $A^{-1} = \frac{adj(A)}{|A|}$

Example

Find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$

Solution

Let A = $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$

Step 1

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix} = 1(18-12) - 1(9-3) + (4-2)$$

$$= 6 - 6 + 2 = 2 \neq 0$$

Step 2

The value of the determinant is non zero

$\therefore A^{-1}$ exists.

Step 3

Let A_{ij} denote the cofactor of a_{ij} in $|A|$

$$A_{11} = Cofactor\ of\ 1 = (-1)^{1+1}\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 18 - 12 = 6$$

$$A_{12} = Cofactor\ of\ 1 = (-1)^{1+3}\begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = -(9 - 3) = -6$$

$$A_{13} = Cofactor\ of\ 1 = (-1)^{1+3}\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 4 - 2 = 2$$

$$A_{21} = Cofactor\ of\ 1 = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -(9 - 4) = -5$$

$$A_{22} = Cofactor\ of\ 2 = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 9 - 1 = 8$$

$$A_{23} = Cofactor\ of\ 3 = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -(4 - 1) = -3$$

$$A_{31} = Cofactor\ of\ 1 = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3 - 2 = 1$$

$$A_{32} = Cofactor\ of\ 4 = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -(3 - 1) = -2$$

$$A_{33} = Cofactor\ of\ 9 = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2 - 1 = 1$$

Step 4

The matrix formed by cofactors of element of determinant $|A|$ is $\begin{pmatrix} 6 & -6 & 2 \\ -5 & 8 & -3 \\ 1 & -2 & 1 \end{pmatrix}$

$\therefore$ adj A = $\begin{pmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{pmatrix}$

Step 5

$$A^{-1} = \frac{adj\ A}{|A|} = \frac{1}{2}\begin{pmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & \frac{-5}{2} & \frac{1}{2} \\ -3 & 4 & -1 \\ 1 & \frac{-3}{2} & \frac{1}{2} \end{pmatrix}$$