PERMUTATION AND COMBINATION

Fundamental Counting Principle

If a first job can be done in ***m*** ways and a second job can be done in ***n*** ways then the total number of ways in which both the jobs can be done in succession is ***m x n.***

For example, consider 3 cities Coimbatore, Chennai and Hyderabad. Assume that there are 3 routes (by road) from Coimbatore to Chennai and 4 routes from Chennai to Hyderabad. Then the total number of routes from Coimbatore to Hyderabad via Chennai is 3 x 4 =12. This can be explained as follows.

For every route from Coimbatore to Chennai there are 4 routes from Chennai to Hyderabad. Since there are 3 road routes from Coimbatore to Chennai, the total number of routes **is 3 x 4 =12.**

The above principle can be extended as follows**.** If there are *n* jobs and if there are m_i ways in which the i^{th} job can be done, then the total number of ways in which all the *n* jobs can be done in succession (1^{st} job, 2^{nd} job, 3^{rd} job… n^{th} job) is given by $m_1 \times m_2 \times m_3 \ldots \times m_n$.

Permutation

Permutation means *arrangement* of things. The word *arrangement* is used, if the order of things is considered. Let us assume that there are 3 plants P_1, P_2, P_3. These 3 plants can be planted in the following 6 ways namely

P_1	P_2	P_3
P_1	P_3	P_2
P_2	P_1	P_3
P_2	P_3	P_1
P_3	P_1	P_2
P_3	P_2	P_1

Each arrangement is called a permutation. Thus there are 6 arrangements (permutations) of 3 plants taking all the 3 plants at a time. This we write as $3P_3$. Therefore $3P_3 = 6$. Suppose out of the 3 objects we choose only 2 objects and arrange them. How many arrangements are possible? For this consider 2 boxes as shown in figure.

I Box	II Box

Since we want to arrange only two objects and we have totally 3 objects, the first box can be filled by any one of the 3 objects, (i.e.) the first box can be filled in 3 ways. After filling the first box we are left with only 2 objects and the second box can be filled by any one of these two objects. Therefore from Fundamental Counting Principle the total number of ways in which both the boxes can be filled is 3 x 2 =6. This we write as $3P_2 = 6$.

In general the number of permutations of *n* objects taking *r* objects at a time is denoted by *nPr*. Its value is given by

$$n\,\mathrm{Pr} = n(n-1)(n-2)\ldots(n-r+1)$$

$$= \frac{n(n-1)(n-2)\ldots.(n-r+1)\times(n-r)(n-r-1)\ldots2.1}{(n-r)(n-r-1)\ldots.2.1}$$

$$i.e \quad n\,\mathrm{Pr} = \frac{n\,!}{(n-r)!}$$

Note: 1

a) $nPn = n\,!$ (b) $nP_1 = n$. (c) $nP_0 = 1$.

Examples:

1. Evaluate $8P_3$

Solution:

$$8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\times7\times6\times5\,!}{5\,!} = 336$$

2. Evaluate $11P_2$

Solution:

$$11\,P_2 = \frac{11!}{(11-2)!} = \frac{11!}{9!} = \frac{11\times10\times9!}{9\,!} = 110$$

3. There are 6 varieties on brinjal, in how many ways these can be arranged in 6 plots which are in a line?

Solution

Six varieties of brinjal can be arranged in 6 plots in $6P_6$ ways.

$$6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6! \qquad [0! = 1]$$

$= 6 \times 5 \times 4 \times 3 \times 2\times1 = 720$.

Therefore 6 varieties of brinjal can be arranged in 720 ways.

4. There are 5 varieties of roses and 2 varieties of jasmine to be arranged in a row, for a photograph. In how many ways can they be arranged, if

(i) all varieties of jasmine together

(ii) All varieties of jasmine are not together.

Solution

i) Since the 2 varieties of jasmine are inseparable, consider them as one single unit. This together with 5 varieties of roses make 6 units which can be arranged themselves in 6! ways.

In every one of these permutations, 2 varieties of jasmine can be rearranged among themselves in 2! ways.

Hence the total number of arrangements required

= 6! x 2! = 720 x 2 = 1440.

ii)The number of arrangements of all 7 varieties without any restrictions =7! = 5040

Number of arrangements in which all varieties of jasmine are together = 1440.

Therefore number of arrangements required = 5040 -1440 = 3600.

Combinatination

Combination means *selection* of things. The word *selection* is used, when the order of thing is immaterial. Let us consider 3 plant varieties V_1, V2 & V_3. In how many ways 2 varieties can be selected? The possible selections are

1) V_1 & V_2
2) V_2 & V_3
3) V_1 & V_3

Each such selection is known as a combination. There are 3 selections possible from a total of 3 objects taking 2 objects at a time and we write $3C_2 = 3$.

In general the number of selections (Combinations) from a total of *n* objects taking *r* objects at a time is denoted by *n Cr.*

Relation between nPr and nCr

We know that

n Pr = nCr x *r!*

(or) $\mathrm{nCr} = \dfrac{\mathrm{nPr}}{\mathrm{r!}}$ ------------(1)

But we know $\mathrm{nPr} = \dfrac{\mathrm{n!}}{(n-r)!}$ ---------(2)

Sub (2) in (1) we get

$$nCr = \frac{n!}{(n-r)!r!}$$

Another formula for *nCr*

We know that *nPr = n. (n-1). (n-2)…(n-r+1)*

$$\therefore \quad nCr = \frac{n.(n-1).(n-2)\,....(n-r+1)}{1.2.3\ ...\ r}$$

Example

1. Find the value of $10C_3$.

Solution:

$$10C_3 = \frac{10\,(9-1)(9-2)}{1.2.3} = \frac{10.\,9.\,8}{1.\,2.3} = 120$$

Note -1

a) $nC_0 = 1$

b) $nC_1 = n$

c) $nCn = 1$

d) $nCr = nCn\text{-}r$

Examples

1**. Find the value of $20C_{18}$**

Solution

We have $20C_{18} = 20C_{20-18} = 20C_2 = \frac{20\times19}{1\times2} = 190$

2. How many ways can 4 prizes be given away to 3 boys, if each boy is eligible for all the prizes?

Solution

Any one prize can be given to any one of the 3 boys and hence there are 3 ways of distributing each prize.

Hence, the 4 prizes can be distributed in $3^4 = 81$ ways.

3. A team of 8 students goes on an excursion, in two cars, of which one can accommodate 5 and the other only 4. In how many ways can they travel?

Solution

There are 8 students and the maximum number of students can accommodate in two cars together is 9.

We may divide the 8 students as follows

Case I: 5 students in the first car and 3 in the second

Case II: 4 students in the first car and 4 in the second

In Case I: 8 students are divided into groups of 5 and 3 in $^8C_3$ ways.

Similarly, in Case II: 8 students are divided into two groups of 4 and 4 in $^8C_4$ ways.

Therefore, the total number of ways in which 8 students can travel is $^8C_3 + {}^8C_4 = 56 + 70 = 126$.

4. How many words of 4 consonants and 3 vowels can be made from 12 consonants and 4 vowels, if all the letters are different?

Solution

4 consonants out of 12 can be selected in $^{12}C_4$ ways.

3 vowels can be selected in $^4C_3$ ways.

Therefore, total number of groups each containing 4 consonants and 3 vowels

$$= {}^{12}C_4 * {}^4C_3$$

Each group contains 7 letters, which can be arranging in 7! ways.

Therefore required number of words = $^{12}4 * {}^4C_3 * 7!$