DIFFERENTIATION

In all practical situations we come across a number of variables. The **variable** is one which takes different values, whereas a **constant** takes a fixed value.

Let x be the independent variable. That means x can take any value. Let y be a variable depending on the value of x. Then y is called the dependent variable. Then y is said to be a function of x and it is denoted by y = f(x)

For example if x denotes the time and y denotes the plant growth, then we know that the plant growth depends upon time. In that case, the function y=f(x) represents the growth function. The rate of change of y with respect to x is denoted by $\frac{dy}{dx}$ and called as the derivative of function y with respect to x.

S.No.	Form of Functions	y=f(x)	$\frac{dy}{dx}$
1.	Power Formula	x^n	$\frac{d(x^n)}{dx} = nx^{n-1}$
2.	Constant	C	0
3.	Constant with variable	Cy	$C\frac{dy}{dx}$
4.	Exponential	e^x	e^x
5.	Constant power x	a^x	$a^x \log a$
6.	Logirthamic	logx	$\frac{1}{x}$
7.	Differentiation of a sum	y = u + v where u and v are functions of x.	$\frac{dy}{dx} = \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$
8.	Differentiation of a difference	y = u – v where u and v are functions of x.	$\frac{dy}{dx} = \frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$
9.	Product rule of differentiation	y = uv, where u and v are functions of x.	$\frac{dy}{dx} = \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
10.	Quotient rule of differentiation	y = $\frac{u}{v}$,	$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$

		where u and v are functions of x.	where $u' = \frac{du}{dx}$, $v' = \frac{dv}{dx}$

Example

1. Differentiate each of the following function $f(x) = 15x^{100} - 3x^{12} + 5x - 46$

Solution

$$f'(x) = 15(100)x^{99} - 3(12)x^{11} + 5(1)x^{0} - 0$$
$$= 1500x^{99} - 36x^{11} + 5$$

2. Differentiate following function $g(t) = 2t^{6} + 7t^{-6}$

Solution

Here is the derivative.

$$g'(t) = 2(6)t^{5} + 7(-6)t^{-7}$$
$$= 12t^{5} - 42t^{-7}$$

3. Differentiate following function $y = 8z^{3} - \frac{1}{3z^{5}} + z - 23$

Solution

$$y = 8z^{3} - \frac{1}{3}z^{-5} + z - 23$$

diff. w.r.to x

$$y' = 24z^{2} + \frac{5}{3}z^{-6} + 1$$

4. Differentiate the following functions.

a) $y = \sqrt[3]{x^{2}}\left(2x - x^{2}\right)$

Solution

$$y = \sqrt[3]{x^{2}}\left(2x - x^{2}\right)$$

$$y = x^{\frac{2}{3}}\left(2x - x^{2}\right)$$

diff y w. r. to x $y' = \frac{2}{3}x^{-\frac{1}{3}}\left(2x - x^2\right) + x^{\frac{2}{3}}\left(2 - 2x\right)$

5. Differentiate the following functions. $f(x) = \left(6x^3 - x\right)\left(10 - 20x\right)$

$$f(x) = \left(6x^3 - x\right)\left(10 - 20x\right)$$

diff f(x) w r to x

$$\begin{aligned} f'(x) &= \left(18x^2 - 1\right)\left(10 - 20x\right) + \left(6x^3 - x\right)\left(-20\right) \\ &= -480x^3 + 180x^2 + 40x - 10 \end{aligned}$$

Derivatives of the six trigonometric functions

$$\frac{d}{dx}\left(\sin(x)\right) = \cos(x) \qquad \frac{d}{dx}\left(\cos(x)\right) = -\sin(x)$$

$$\frac{d}{dx}\left(\tan(x)\right) = \sec^2(x) \qquad \frac{d}{dx}\left(\cot(x)\right) = -\csc^2(x)$$

$$\frac{d}{dx}\left(\sec(x)\right) = \sec(x)\tan(x) \qquad \frac{d}{dx}\left(\csc(x)\right) = -\csc(x)\cot(x)$$

Example

1. Differentiate each of the following functions.

$$g(x) = 3\sec(x) - 10\cos(x)$$

Solution We'll just differentiate each term using the formulas from above.

$$\begin{aligned} g'(x) &= 3\sec(x)\tan(x) - 10\left(-\sin(x)\right) \\ &= 3\sec(x)\tan(x) + 10\sin(x) \end{aligned}$$

2. Differentiate each of the following functions $y = 5\sin(x)\cot(x) + 4\csc(x)$

Here's the derivative of this function.

$$\begin{aligned} y' &= 5\cos(x)\cot(x) + 5\sin(x)\left(-\csc^2(x)\right) - 4\csc(x)\cot(x) \\ &= 5\cos(x)\cot(x) - 5\csc(x) - 4\csc(x)\cot(x) \end{aligned}$$

Note that in the simplification step we took advantage of the fact that

$$\csc(x) = \frac{1}{\sin(x)}$$

to simplify the second term a little.

3. Differentiate each of the following functions $P(t) = \dfrac{\sin(t)}{3 - 2\cos(t)}$

In this part we'll need to use the quotient rule.

$$\begin{aligned} P'(t) &= \frac{\cos(t)\left(3 - 2\cos(t)\right) - \sin(t)\left(2\sin(t)\right)}{\left(3 - 2\cos(t)\right)^2} \\ &= \frac{3\cos(t) - 2\cos^2(t) - 2\sin^2(t)}{\left(3 - 2\cos(t)\right)^2} \end{aligned}$$